HEIN – MODEL

Description

The HEIN-model is an elasto-viscoplastic creep law to simulate porous materials, e. g. crushed salt. The elastic behavior is determined by porosity depending elastic properties, with low bulk and shear modulus at the beginning of the compaction, reaching the values of compacted material at total compaction. The viscoplastic part is a combined hydrostatic – deviatoric law.

A more detailed description of the HEIN model can be found in

- Hein, H.-J. „Ein Stoffgesetz zur Beschreibung des thermomechanischen Verhaltens von Salzgranulat". Dissertation, RWTH Aachen, 1991.
- Lerch, C., „Abschlußbericht zum Projekt BAMBUS. Teilprojekt Comparative Study on Crushed Salt (CS)^2". Deutsche Gesellschaft zu Bau und Betrieb von Endlagern für Abfallstoffe mbH (DBE), Peine, 1999.

Mathematical formulation

The total strain rate can be separated into a thermoelastic and a viscoplastic part:

$$\underline{\underline{\dot{e}}} = \underline{\underline{\dot{e}}}_{th-el} + \underline{\underline{\dot{e}}}_{vp}$$

The thermally-induced stresses will be neglected here. The elastic part is given by Hooke's law

$$\underline{\underline{s}} = K \cdot \underline{\underline{e}}_{el}^{vol} + 2G \cdot \underline{\underline{e}}_{el}^{dev}$$

where the elastic deformation tensor is divided into a volumetric and a deviatoric part

$$\underline{\underline{e}}_{el} = \underline{\underline{e}}_{el}^{vol} + \underline{\underline{e}}_{el}^{dev}$$

The viscoplastic deformation rate is calculated as

$$\underline{\underline{\dot{e}}}_{vp} = A \cdot e^{-\frac{Q}{RT}} \cdot \left(h_1 p^2 + h_2 q^2\right)^n \cdot \left(\frac{1}{3} h_1 p \underline{\underline{I}} + h_2 \underline{\underline{S}}\right)$$

with

$$P_{\max} = \frac{c_4}{c_5}\left(\left(\frac{1-h}{1-h_0}\right)^{c_5} - 1\right)$$

$$h_1 = \frac{1 - c_1 \cdot e^{c_2 h} \cdot c_3^2}{P_{\max}^2}$$

$$h_2 = c_6 + c_7 \cdot h_1$$

$$\underline{\underline{S}} = \underline{\underline{s}} - \frac{1}{3} s_{ii} \cdot \underline{\underline{I}}$$

$$p = \frac{s\,ii}{3} \quad (hydrostatic\ pressure)$$

$$q = \sqrt{2}\sqrt{J_2}$$

$\underline{\underline{I}}$ is the unit tensor, J_2 the second stress invariant. $\underline{\underline{S}}$ is the stress deviator

$$\underline{\underline{S}} = \underline{\underline{s}} - \frac{1}{3} s_{ii} \cdot \underline{\underline{I}}$$

Bulk modulus *K* and shear modulus *G* depend on the actual porosity η

$$K = K_0 \cdot e^{-c_K \cdot \eta \cdot \left(\frac{1-\eta_0}{1-\eta}\right)}$$

$$G = G_0 \cdot e^{-c_G \cdot \eta \cdot \left(\frac{1-\eta_0}{1-\eta}\right)}$$

where K_0 and G_0 are the bulk and shear modulus at full compaction (η=0), respectively. The actual porosity η is derived from the initial porosity η_0 and the volumetric deformation e^{vol}

$$\eta = \frac{\eta_0 + e^{vol}}{1 + e^{vol}}$$

In some cases it may be necessary to take the higher-order elements into account for the calculation of the volumetric deformation. Therefore, a switch `evol_complete=(0/1)` has been implemented, such that

$$e^{vol} = \begin{cases} \frac{1}{3} e_{ii} & evol\_complete = 0 \\ \det\left|\underline{\underline{F}}\right| - 1 & evol\_complete \neq 0 \end{cases}$$

$\underline{\underline{F}}$ is the deformation gradient tensor. For `evol_complete≠0`, the volumetric deformation is calculated as

$$e^{vol} = e_x + e_y + e_z + e_x e_y + e_x e_z + e_y e_z + e_x e_y e_z - e_x e_{yz}^2 - e_y e_{xz}^2 - e_z e_{xy}^2 + 2 e_{xy} e_{xz} e_{yz}$$

Input parameters

Property name	Description	Description in literature	Typical data set (exemplary)	Modifiable
`bulk`	Bulk modulus	K	$1.087 \cdot 10^3$	X
`K_0`	Bulk modulus at total compaction	K_0	$1.812 \cdot 10^4$	X
`shear`	Shear modulus	G	$5.906 \cdot 10^3$	X
`G_0`	Shear modulus at total compaction	G_0	$9.843 \cdot 10^3$	X
`young`	Young's modulus	E		X
`E_0`	Young's modulus at total compaction	E_0		X
`Poisson`	Poisson's ratio	ν		X
`eta_0`	Initial porosity	η_0	0.3103	X
`struct_factor`	constant	A	$1.41891 \cdot 10^{-18}$	X
`act_energy`	Activation energy	Q	Q/R=17531.4481	X

Property name	Description	Description in literature	Typical data set (exemplary)	Modifiable
gas_C	Gas constant	R		X
temperature	Temperature	T		X
C_1	Constant	c_1	2.8696804	X
C_2	Constant	c_2	-0.15336035	X
C_3	Constant	c_3	0.59	X
C_4	Constant	c_4	0.12617203	X
C_5	Constant	c_5	1.36150609	X
C_6	Constant	c_6	6.70060765	X
C_7	Constant	c_7	0.88785261	X
C_8	Initial value for P_{max} P_{max}=c_8 for $\eta=\eta_0$	c_8	0.001	X
n_stress	Stress exponent	n	8.94364086	X
evol	Volumetric deformation	ε^{vol}		X
c_k	Compaction parameter for *K*	c_k		X
poros	Actual porosity	η		
c_G	Compaction parameter for *G*	c_G		X
evol_complete	Switch for taking higher order elements into account for the calculation of the deformation			X
eps_11		ε_{11}		
eps_22		ε_{22}		
eps_33		ε_{33}		
eps_12		ε_{12}		
eps_13		ε_{13}		
eps_23		ε_{23}		
rho_0		ρ_0		
vol_0				
evol_0				X

Practical hints

- The creep option is needed to use the HEIN constitutive model.
- K, K_0, G, G_0, h, c_K *and* c_G are not independent from each other. The parameters K_0, G_0, c_K, c_G and η_0 have to be given, from which the actual elastic parameter are determined. The user has to take care that the input parameter are consistent.

Included documents / files

All four input files are simulating a quarter model of a borehole. Inside the borehole, the

Name	Type	Description
Hein_small.dat	Flac^{3D} – Input-File	Single-element test, small strain
Hein_large.dat	Flac^{3D} – Input-File	Single-element test, large strain
hein.dll	Dynamic Link Library	Contains the Hein constitutive model

Contact address

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